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Design research on learning lines and angles through problem-based learning with GeoGebra

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ABSTRACT

Lines and angles are foundational topics in junior high school geometry. However, international research consistently shows that students struggle to translate visual representations into formal geometric reasoning. These difficulties are often exacerbated by conventional, teacher-centred instruction that lacks dynamic visualisation, leaving students passive and hesitant to articulate their reasoning. This study aimed to design, implement, and iteratively refine a learning trajectory integrating Problem-Based Learning (PBL) with GeoGebra to improve Grade VII students' conceptual understanding of lines and angles. Employing a design research approach, the study followed three phases: preparation, design experiment, and retrospective analysis. The design experiment involved two implementation cycles at a public junior high school in Serang Regency, Indonesia. Cycle 1 involved a small-group trial with five students, while Cycle 2 was conducted with a full class of 33 students. The Hypothetical Learning Trajectory (HLT) consisted of three sequential activities: identifying angle types through clock-hand movements, exploring relationships between pairs of angles through tangram arrangements, and investigating parallel lines cut by a transversal using road intersection contexts. The findings indicate that the refined learning trajectory was feasible for classroom implementation and supported a shift from visual guessing towards more explicit use of angle definitions and relationships. In Cycle 2, the proportion of students in the high category increased from 9% to 45%, while the proportion in the low category decreased from 61% to 18%. This study contributes a topic-specific, classroom-tested learning trajectory and offers a practical model for teaching lines and angles in similar educational contexts.

KEYWORDS

Design research; hypothetical learning trajectory; problem-based learning; GeoGebra; lines and angles

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INTRODUCTION

Geometry is a foundational domain in mathematics education worldwide because it uniquely bridges abstract mathematical reasoning and spatial visualisation. Among the various topics in geometry, lines and angles occupy a particularly important position at the junior high school level, as they serve as conceptual gateways to more advanced topics such as polygons, triangles, and three-dimensional figures. However, international research has consistently shown that lines and angles are among the most difficult topics for students to master. Karadeniz and Hodanci (2025) found that seventh-grade students in Turkey could identify only 33% of angles in real-world visual contexts and struggled to connect angle concepts with everyday situations. This finding indicates that difficulties in translating visual representations into formal geometric reasoning are not confined to a single educational system. Similar challenges have also been documented in studies of geometry instruction at the secondary level, where

students struggle to visualise abstract relationships and apply them in problem-solving contexts (Nuralam et al., 2024).

These difficulties are often exacerbated by conventional, teacher-centred instruction that lacks dynamic instructional media for visualising geometric relationships. Under such conditions, students tend to become passive and hesitant to articulate their reasoning, a pattern observed across diverse educational contexts. In the Indonesian context, for instance, data from the 2019 National Computer-Based Examination (UNBK) in Serang Regency revealed that only 34.88% of junior high school students achieved the minimum competency standard in geometry, including lines and angles (Kementerian Pendidikan dan Kebudayaan, 2019). This pattern has been linked, in similar local contexts, to mathematics anxiety and low confidence when dealing with abstract tasks (Hadi et al., 2020). More broadly, Salsabila et al (2023) highlighted that students' ability to reason critically and creatively depends heavily on their understanding of foundational mathematical concepts. Taken together, these findings underscore the urgent need for an instructional shift that addresses both visualisation challenges and the limited opportunities for active reasoning in geometry classrooms.

An effective learning environment for this topic should engage students actively through meaningful problems while also supporting dynamic visualisation. Problem-Based Learning (PBL) addresses the first need by using real-world problems as the starting point for learning and requiring students to analyse problems collaboratively, propose strategies, and reflect on their results. These processes are closely associated with critical and creative thinking (Haryati & Wangid, 2023; Putri et al., 2023). GeoGebra addresses the second need by enabling students to construct and manipulate geometric objects, measure angles, and observe in real time how changes in one element affect others (Nuryati & Reflina, 2023). International studies have shown that GeoGebra-assisted instruction can enhance conceptual understanding, increase motivation, and help students develop stronger geometric reasoning across various secondary school contexts (Iklimah et al., 2023; Nuralam et al., 2024; Priatna & Arsani, 2019). Together, PBL and GeoGebra offer a coherent and promising instructional approach: PBL drives inquiry through contextual problems, while GeoGebra provides the interactive visual support needed to reduce cognitive load and deepen geometric reasoning.

Previous studies have established important foundations by showing that visual-spatial approaches can improve students' understanding of angle concepts (Bustang, 2013), contextual tasks can support the understanding of lines and angles (Widiawati et al., 2018), and PBL supported by GeoGebra can outperform PBL alone in improving conceptual understanding of

geometric topics such as polyhedra (Khurniati et al., 2023). Furthermore, design research approaches using a Hypothetical Learning Trajectory (HLT) have been shown to help students overcome specific learning obstacles in geometry by sequencing tasks according to anticipated student thinking (Kuncoro et al., 2023). However, none of these studies has yet provided a design-research-based learning trajectory that integrates PBL and GeoGebra specifically for Grade VII students learning lines and angles, is grounded in documented student difficulties, and is described in sufficient detail to serve as a practical classroom reference. Accordingly, this study aims to design, implement, and iteratively refine such a learning trajectory in a public junior high school in Serang Regency. Its novelty lies not simply in combining PBL and GeoGebra, but in developing a topic-specific, classroom-tested learning trajectory that maps GeoGebra-supported contextual activities onto PBL phases in direct response to students' difficulties in identifying angle relationships and providing mathematical justifications. Although the study is situated in the Indonesian context, the resulting trajectory offers a theoretically informed and practically applicable model that may inform geometry instruction in other educational settings facing similar challenges.

METHODS

This study employed a design research approach, following the tradition established by Gravemeijer and Cobb (2006), to develop, implement, and refine a learning design for teaching lines and angles through a Problem-Based Learning (PBL) model supported by GeoGebra. Design research was selected because its primary aim was to generate a transferable instructional product, specifically a Hypothetical Learning Trajectory (HLT), that could inform both theory and practice beyond the immediate research context. In this study, the HLT was structured as a learning design comprising three essential components: learning goals, a planned sequence of instructional activities, and conjectures about students' thinking processes (Magdalena et al., 2021). The instructional activities comprised three stages mapped onto the PBL phases: (1) identifying angle types through clock-hand movements, (2) exploring supplementary, complementary, and vertically opposite angles through tangram arrangements, and (3) investigating angle relationships formed by parallel lines cut by a transversal through road-intersection contexts. These activities were sequenced in accordance with the hierarchical structure of the topic of lines and angles specified in the Indonesian 2013 Curriculum (Kementerian Pendidikan dan Kebudayaan, 2017), This sequencing reflected the principle that students need to develop foundational conceptual understanding before engaging in more complex relational reasoning (Baiduri et al., 2021).

The study was conducted at a public junior high school in Serang Regency, Indonesia, involving Grade VII students who were studying lines and angles as part of the regular mathematics curriculum. Two implementation cycles were conducted across two intact classes to systematically test and refine the HLT, contributing to the development of a Local Instruction Theory for teaching lines and angles at the Grade VII level. Cycle 1 involved a small group of five students from one class (Group 1), whereas Cycle 2 involved a full class of 33 students from a different class (Group 2). The selection of five students for Cycle 1 was intentional and methodologically motivated. As Gravemeijer and Cobb (2006) indicated, a small-group trial allows researchers to test initial conjectures in the HLT, closely observe students' responses, and identify weaknesses in task design before wider classroom implementation. The two groups were not treated as equivalent comparison groups; rather, they served different purposes in the iterative testing and refinement of the instructional design.

The research followed three main phases: preparation, design experiment, and retrospective analysis. During the preparation phase, the researcher analysed the learning needs and characteristics of Grade VII students. As part of this needs analysis, a diagnostic pre-test on the basic concepts of lines and angles was administered to all 38 students across Groups 1 and 2 to establish a baseline profile. The researcher also reviewed relevant literature on PBL and GeoGebra in mathematics education and prepared the instructional materials, including lesson plans, student worksheets integrating contextual problems and GeoGebra activities, and pre-test and post-test instruments.

During the design experiment phase, Cycle 1 was used to examine the clarity of the worksheets, the suitability of the problems for students' prior knowledge, the feasibility of the time allocation, and students' initial responses to using GeoGebra. Based on these findings, the design was revised by, for example, adjusting the wording of the worksheets, refining the GeoGebra instructions, and reorganising discussion time. The revised design was then implemented in Cycle 2, during which the researcher focused on student participation in PBL activities and changes in conceptual understanding throughout the cycle. Any improvements observed in Cycle 2 were interpreted cautiously as evidence of how the refined design functioned in that classroom context, rather than as proof that the revisions alone caused the improvements. Classroom activities were documented through observations, field notes, and visual recordings.

Retrospective analysis was subsequently conducted by comparing the HLT with the actual learning trajectory observed in the classroom. The analysis focused on the intended

learning goals, the sequence of PBL and GeoGebra-supported activities, and the conjectures about students' thinking. It examined the extent to which the enacted lessons supported students' conceptual understanding of angle relationships and identified typical misconceptions and difficulties that emerged during group work and classroom discussions (Kuncoro et al., 2023). The findings were used to determine which elements of the instructional design required revision for the subsequent cycle.

Data were collected using multiple instruments. Classroom observations, guided by a 12-question observation sheet, documented student engagement, group interaction, and the use of GeoGebra. Semi-structured interviews were conducted in Indonesian after the lessons with students representing different levels of initial understanding to explore their experiences with PBL and GeoGebra. Field notes captured contextual details, such as the classroom atmosphere, technical difficulties, and the researcher's reflections. The pre-test and post-test, each consisting of five open-ended items with a maximum score of 20, were used to measure conceptual understanding before and after the lessons. The items assessed students' ability to restate concepts, classify angle relationships, and explain concepts in their own words. Because the items also required students to analyse diagrams and justify angle relationships, they provided additional, although limited, evidence of mathematical reasoning. Critical and creative thinking were examined more extensively through students' participation in PBL discussions, interview responses, and written work in the worksheets. Test scores were categorised into three levels: low (0–6), medium (7–13), and high (14–20).

Data were analysed using complementary qualitative and quantitative procedures integrated during the retrospective analysis to produce findings that were both locally grounded and methodologically rigorous (Kuncoro et al., 2023). Quantitative data from the tests were summarised descriptively to identify areas of improvement and aspects that remained difficult for students. Qualitative data were analysed descriptively to identify patterns in student engagement, GeoGebra use, problem-solving strategies, and misconceptions. The two types of data were integrated at the interpretation stage: quantitative results indicated the extent of learning changes, while qualitative findings helped explain how specific design features supported or hindered those changes. The credibility of the findings was strengthened through triangulation across data sources, methods, and cycles by comparing observation records, field notes, students' worksheet responses, test results, interview data, and evidence from classroom discussions.

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RESULT AND DISCUSSION

Preparation for the Experiment

The preparation phase yielded three main outcomes: a diagnostic profile of students' understanding, a synthesis of relevant literature that informed the design principles, and a prototype learning design integrating PBL with GeoGebra for Grade VII students. A diagnostic pre-test on the basic concepts of lines and angles was administered to 38 students from Groups 1 and 2. The results are presented in Table 1.

Table 1. Item-Level Pre-test Performance (N = 38, Maximum Score = 4 per item)

Item	Indicator	Mean Score
1	Identifying and representing angle types (acute, right, obtuse, straight)	1.39
2	Classifying vertically opposite angles at intersecting lines	0.61
3	Applying the supplementary angle concept to find unknown angles	0.63
4	Classifying angle pairs formed by parallel lines and a transversal	0.32
5	Integrating multiple angle relationships to determine all angles	0.26

Table 1 reveals a clear gradient of difficulty across the five test items. Students were relatively more successful in identifying and representing basic angle types (Item 1, mean = 1.39), although the mean score remained below half of the maximum possible score, indicating that even the most accessible concept had not been fully mastered. Performance declined substantially on items requiring relational reasoning, including classifying vertically opposite angles (Item 2, mean = 0.61), applying the concept of supplementary angles (Item 3, mean = 0.63), and identifying angles formed by parallel lines and a transversal (Item 4, mean = 0.32). The lowest performance was observed on Item 5 (mean = 0.26), which required students to integrate multiple angle relationships within a single diagram. Most students either left this item unanswered or provided inconsistent answers. This pattern indicates that students' difficulties were not limited to a single concept but reflected a broader challenge in progressing from isolated angle recognition to relational and integrative reasoning.

A review of three key studies informed the design principles. Bustang (2013) showed that visual-spatial activities support the development of robust understanding of angle concepts, Widiawati et al (2018) demonstrated that realistic contexts promote students' understanding of lines and angles, and Khurniati et al (2023) found that PBL supported by GeoGebra outperforms PBL alone in improving conceptual understanding. Building on these findings, the present study developed a coherent learning trajectory specifically for teaching lines and angles to Grade VII students.

Based on these inputs, the preparation phase produced a prototype learning design in the form of an initial Hypothetical Learning Trajectory (HLT), consisting of three sequential

activities progressing from basic angle classification to more complex angle relationships. Each activity was aligned with the main phases of PBL: problem orientation, group investigation, discussion and presentation, and reflection (Putri et al., 2023). The prototype also specified the learning objectives, the contextual problem used to initiate inquiry, the role of GeoGebra in supporting exploration, and conjectures about students' thinking, together with the teacher support that might be required. According to Magdalena et al (2021), specifying these elements in advance enables the prototype to function both as a plan of action and as a predictive tool, allowing the researcher to anticipate likely misconceptions and prepare appropriate scaffolding.

The first activity focused on identifying and classifying angle types through the familiar context of clock-hand movements. It aimed to help students recognise acute, right, obtuse, and straight angles in everyday situations. Students were introduced to the problem through questions about the angles formed by the hour and minute hands at different times. They then used GeoGebra to model clock-hand positions and measure the angles formed at selected times, such as 2:00, 3:30, 5:00, and 8:20. Because students were expected to rely on their visual perception of the "space" between the hands rather than on precise measurement, the design included explicit GeoGebra instructions and guiding questions that connected visual observations to the 90° and 180° benchmarks.

The second activity addressed angle relationships through tangram arrangements, aiming to help students identify and explain supplementary, complementary, and vertically opposite angles. Students used GeoGebra to manipulate tangram pieces, create straight-angle, right-angle, and intersecting-angle configurations, and measure the resulting angles. They then discussed patterns and reflected on why some pairs of angles sum to 90° or 180°, whereas others have equal measures. Because students were expected to focus primarily on numerical measurements, teacher prompts were prepared to encourage them to name each relationship, explain why it held, and connect the measured values to the underlying geometric configurations.

The third activity extended the learning trajectory to the context of two parallel roads intersected by a transversal. It aimed to help students identify and apply corresponding, alternate, and same-side angle relationships to determine unknown angle measures. Students used GeoGebra to model the situation, identify the positions of angles, measure the angles formed, and investigate relationships involving corresponding angles, alternate interior and exterior angles, and same-side interior and exterior angles. Because students were expected to confuse interior and exterior positions or rely on measurements without providing explanations,

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teacher prompts were prepared to help them mark angles accurately, distinguish between positional relationships, and articulate their reasoning.

Taken together, the three activities formed a coherent prototype that directly addressed the diagnostic findings. Activity 1 began with visual classification tasks in the familiar context of clock-hand movements, using GeoGebra to bridge visual intuition and formal angle benchmarks. Activity 2 required students to explain why pairs of angles are supplementary, complementary, or vertically opposite, using tangram configurations and GeoGebra measurements as evidence rather than relying solely on estimation. Activity 3 was deliberately positioned last so that students could approach the more complex system of parallel lines with the foundational knowledge and reasoning strategies developed through the two preceding activities. GeoGebra was integrated into each activity not merely as a measurement tool but also as a means for students to test predictions, observe invariant relationships, and connect visual exploration with formal geometric reasoning. As illustrated in Figure 1, the design progressed from familiar visual contexts to increasingly complex relational reasoning.

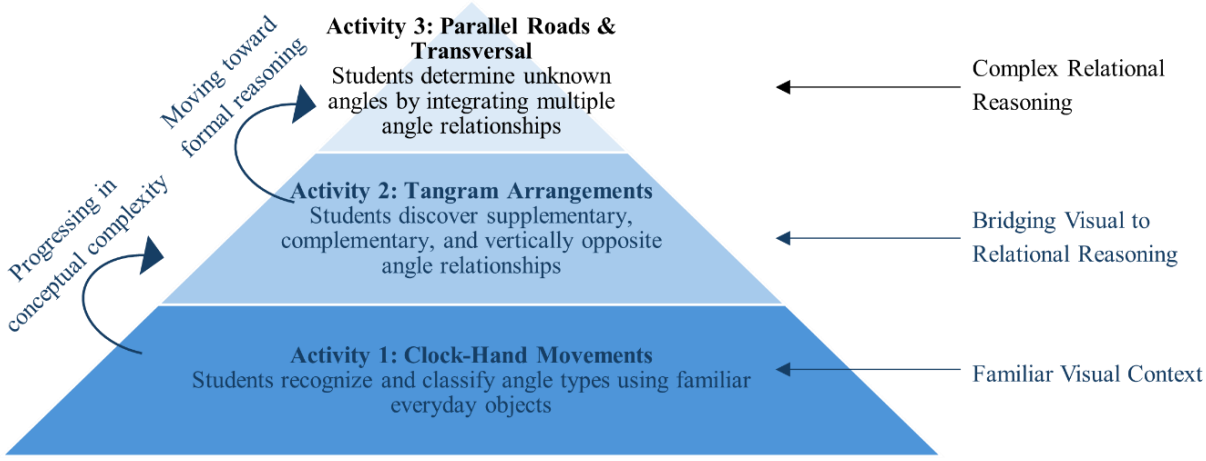


Figure 1. Hypothetical Learning Trajectory from Visual to Relational Reasoning

Design Experiment

In Cycle 1, all three activities were implemented as planned, although several practical and conceptual issues emerged. Classroom observations showed that students were generally engaged, particularly when working with GeoGebra. However, their levels of participation varied according to their technical confidence and prior familiarity with the software.

In Activity 1 (clock-hand context), students initially classified angles based on visual intuition rather than measurement. Several students experienced difficulties using GeoGebra's angle tool. However, through peer discussion and demonstrations by the researcher, they gradually shifted towards quantitative reasoning. For example, one student initially guessed that the angle at 3:30 was obtuse but revised this classification after measuring the angle as 75°,

recognising that it was acute.

In Activity 2 (tangram context), students' exploration initially stalled because all the tangram pieces were the same colour, which drew their attention towards matching shapes rather than analysing angles. Three of the five students initially wrote arbitrary values when calculating an unknown supplementary angle. However, after being prompted to recall the measure of a straight angle, they began applying the 180° rule correctly and explaining angle relationships using both measurement results and geometric positions.

In Activity 3 (parallel-road intersection), students showed genuine interest in the real-world context. However, procedural errors when using GeoGebra resulted in inaccurate angle measurements until the researcher demonstrated the correct point-selection procedure. Classroom dialogue and post-activity interviews indicated that students found it easier to reason from measurement towards justification than to move from definitions towards predictions. One student noted that GeoGebra made it possible to see which angles were equal before attempting to explain why.

Table 2. Distribution of Students by Understanding Category in Cycle 1 (N = 5)

Category	Score Range	Pre-test (f)	Post-test (f)
Low	0–6	3	0
Medium	7–13	1	3
High	14–20	1	2

Table 2 illustrates the changes in students' understanding between the pre-test and post-test in Cycle 1. Initially, three students were classified in the low category, while only one reached the high category. Following the three PBL and GeoGebra sessions, no students remained in the low category, two reached the high category, and three were classified in the medium category. These changes suggest that the initial design had the potential to support conceptual development, particularly by helping students move beyond visual guessing towards applying basic angle definitions. However, the persistent difficulties observed in items requiring students to classify angle relationships and reason across multiple angle concepts simultaneously indicated that the design required further refinement. Taken together, the classroom observations, interview responses, and test-score patterns identified four specific problems that directly informed the revisions made before Cycle 2, as summarised in Table 3.

Table 3. Summary of Design Revisions from Cycle 1 to Cycle 2

Issues Identified in Cycle 1	Revisions Implemented for Cycle 2
All tangram pieces were the same colour, causing students to focus on matching shapes rather than analysing angles.	The tangram pieces were colour-coded according to relationship type. An initial worked example was added, and a dedicated "Reason" column was included in the worksheet table.

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Issues Identified in Cycle 1	Revisions Implemented for Cycle 2
Procedural errors when using GeoGebra in Activities 1 and 3, particularly selecting points in the wrong order, disrupted the learning process.	Step-by-step GeoGebra instructions were made more explicit. A simpler example, such as measuring the angle at 3:00, was introduced before students attempted more complex cases.
Students' reasoning consistently followed an inductive pattern, moving from measurement towards justification, rather than the deductive pattern anticipated in the HLT.	The task sequence in Activity 3 was restructured so that students first identified angle pairs based on their positions, predicted the relationships, measured the angles using GeoGebra, and then reflected on whether the results were consistent with the relevant definitions.
The larger class size in Cycle 2 required more structured group collaboration.	Group roles were formally assigned, including a GeoGebra navigator, a recorder, a questioner responsible for focusing on definitions and reasoning, and a presenter.

In Cycle 2, students' interactions with GeoGebra were noticeably more fluent, reflecting the impact of the revised procedural instructions and colour-coded tangram layout. In Activity 1, most students used the angle tool independently and compared the measured angles with the 90° benchmark without prompting. Two reasoning strategies were evident: visual classification based on the perceived "width" of an angle and formal classification based on its degree measure. Group presentations helped consolidate these approaches into explicit definitions of acute, right, and obtuse angles.

In Activity 2, the colour-coded pieces and locked grid eliminated the confusion about shape matching observed in Cycle 1. Seven of the nine groups correctly solved the problems involving supplementary, complementary, and vertically opposite angles with minimal guidance. Students explained their answers using both measurement results and geometric reasoning. For example, they noted that supplementary angles sum to 180° because the pieces form a straight line.

In Activity 3, most groups correctly identified corresponding, alternate, and same-side angle pairs and used GeoGebra to verify their predictions. Some students dragged the transversal to confirm that the angle relationships remained valid regardless of its position. A few groups still confused same-side and alternate angle positions, but inter-group discussions helped resolve these errors. Post-activity interviews confirmed that GeoGebra's dynamic features made the angle properties more convincing because students could test the relationships themselves.

Table 4. Distribution of Students by Understanding Category in Cycle 2 (N = 33)

Category	Score Range	Pre-test (f)	Post-test (f)
Low	0–6	20	6
Medium	7–13	10	12
High	14–20	3	15

Table 4 shows that, before the intervention, only three students were in the high category, whereas 20 were in the low category. Following the revised PBL and GeoGebra lessons, 15 students reached the high category, 12 were classified in the medium category, and only six remained in the low category. This pattern indicates substantial improvement within Cycle 2, particularly in moving students out of the low category. When considered alongside the findings from Cycle 1, the results suggest that the revised design remained productive in a larger and more heterogeneous classroom. Although no students remained in the low category at the end of Cycle 1, that cycle involved only a small exploratory group receiving intensive guidance. In contrast, Cycle 2 was conducted in a full class of 33 students. In this setting, the revised design supported a larger proportion of students in reaching the high category while substantially reducing the number in the low category from 20 to six. These findings indicate that the revised learning trajectory was workable in a more typical classroom context, although some students still required additional support in classifying angle relationships and providing mathematical justifications.

Retrospective Analysis

The retrospective analysis indicates that the refined PBL and GeoGebra learning trajectory supported a shift in students' reasoning from visual guessing and isolated computation towards more explicit use of angle definitions and relationships. Across the two cycles, students increasingly justified their answers by referring to supplementary, complementary, vertically opposite, corresponding, and same-side angle relationships rather than relying solely on their visual perception of angle size. This finding is consistent with earlier design research suggesting that understanding of angles develops more meaningfully through structured visual and contextual experiences than through formal explanations alone (Bustang, 2013).

The findings also extend previous research. Whereas Bustang (2013) worked with primary school students and showed that contextual visual tasks could help learners develop initial concept images of angles, the present study suggests that some Grade VII students still held incomplete or unstable concept images when they began the lessons. For example, three students assumed that all angles at an intersection point were equal in size, reflecting confusion between visual symmetry and geometric relationships. Several others incorrectly treated supplementary angles as equal rather than recognising that their measures sum to 180°, while some provided arbitrary angle values when asked to calculate unknown measures. These patterns suggest that difficulties with angle concepts are not necessarily resolved during earlier

schooling and may persist into junior high school unless instruction deliberately reconnects visual experiences, definitions, and formal properties. The contribution of this study, therefore, lies in demonstrating how this developmental process can be supported at the Grade VII level through a structured PBL and GeoGebra learning trajectory.

The role of context was also central to the learning trajectory. The clock-hand, tangram, and parallel-road tasks provided students with accessible starting points for discussing angles before formal terminology was introduced or refined. This is consistent with Widiawati et al (2018), who found that contextual activities helped Grade VII students understand lines and angles through meaningful situations. However, the present study differs in that the contexts were organised into a progressive sequence: from classifying basic angle types, to recognising angle-pair relationships, and finally to reasoning about systems of angles formed by parallel lines and a transversal. This suggests that context is most productive not merely as a means of motivation but also as part of a carefully sequenced learning trajectory that supports gradual mathematization.

The study also clarifies the role of GeoGebra within this trajectory. Consistent with Khurniati et al (2023), GeoGebra supported conceptual understanding by making angle relationships visible and testable. Students used its measurement and dragging features to verify predictions, identify invariant relationships, and connect diagrams with formal geometric properties. For example, during Cycle 2, some students used the dynamic dragging feature to confirm that corresponding angles remained equal as the transversal was moved. However, the findings also show that technology alone was insufficient. In Cycle 1, students often misused GeoGebra tools or treated measurements as answers in themselves rather than as evidence requiring interpretation. In contrast to effectiveness studies that primarily report improvements in test scores (e.g., Khurniati et al., 2023), the present study provides a more process-oriented account by showing that the educational value of GeoGebra depends on task design, scaffolding, and classroom discussion.

Methodologically, the study demonstrates the value of design research and HLT refinement. In line with Gravemeijer and Cobb (2006), comparing the Hypothetical Learning Trajectory (HLT) with the actual learning trajectory enabled the researcher to identify points at which the initial design did not fully align with students' thinking. A systematic retrospective analysis of Cycle 1 data, drawing on classroom observation records, field notes, and item-level analyses of pre-test and post-test responses, revealed three specific mismatches between the intended and actual learning trajectories. First, students tended to guess angle measures in the

tangram tasks rather than apply angle relationships. This pattern was confirmed by classroom dialogue, in which students proposed arbitrary values before receiving guiding prompts. Second, students made procedural errors when using GeoGebra, particularly when selecting points in the wrong order, resulting in inaccurate measurements. Third, students' reasoning consistently followed an inductive pattern, moving from measurement data towards justification, rather than the deductive pattern anticipated in the HLT. These findings informed the redesign process. The revisions introduced in Cycle 2, including colour-coded tangram pieces, clearer procedural instructions for GeoGebra, and task sequences requiring students to predict angle relationships before measuring, were followed by more consistent participation and greater use of geometric definitions. In this respect, the study contributes not only a workable instructional design but also a locally grounded account of how an HLT can be systematically improved through retrospective analysis and iterative revision.

The findings also provide a more nuanced perspective on the generally positive results reported in previous studies. Although earlier research has shown that contextual learning and GeoGebra-assisted PBL can improve learning outcomes (Widiawati et al., 2018; Khurniati et al., 2023), the present study shows that conceptual gains remain uneven. Even in Cycle 2, some students continued to confuse same-side and alternate angles or relied heavily on measured values before they could articulate the relevant definitions. This pattern is consistent with Nuralam et al (2024), who found that even when students used GeoGebra to construct and visualise geometric problems, they still required guided synthesis of prior knowledge to develop mathematical reasoning. These findings suggest that GeoGebra functions most effectively as a cognitive tool when its use is supported by deliberate pedagogical scaffolding rather than when it is offered as a stand-alone visualisation aid. Contextual activities and digital tools are necessary but not sufficient; teacher questioning, structured comparisons of strategies, and explicit attention to definitions remain essential for helping students progress from empirical observation to mathematical justification.

Building on this argument, an effective PBL and GeoGebra design should integrate scaffolding mechanisms throughout the learning trajectory rather than treat them as supplementary support. First, GeoGebra exploration should be consistently accompanied by reasoning prompts that ask students to predict, justify, and reflect on their measurements. In this way, empirical observation becomes the starting point for mathematical argumentation rather than its endpoint (Nuralam et al., 2024). who found that even when students used GeoGebra to construct and visualise geometric problems, they still required guided synthesis

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Building on this argument, an effective PBL and GeoGebra design should integrate scaffolding mechanisms throughout the learning trajectory rather than treat them as supplementary support. First, GeoGebra exploration should be consistently accompanied by reasoning prompts that ask students to predict, justify, and reflect on their measurements. In this way, empirical observation becomes the starting point for mathematical argumentation rather than its endpoint (Bustang, 2013). Furthermore, the analysis focused primarily on students' conceptual understanding and gave limited attention to affective and metacognitive dimensions, such as motivation, confidence, and self-regulated reasoning. The contextual problems incorporated into the trajectory also represented only a small subset of the real-world situations in which lines and angles appear. In light of these limitations, future research should test and refine the trajectory in other classrooms involving different teachers and student populations, conduct additional design cycles to develop a more stable LIT, investigate the effects of the PBL and GeoGebra trajectory on affective and metacognitive aspects of learning, and expand the range of contextual problems to include situations more closely connected to students' everyday experiences.

CONCLUSION

This study aimed to design and iteratively refine a Problem-Based Learning (PBL) trajectory supported by GeoGebra to improve Grade VII students' conceptual understanding of lines and angles. The findings show that the resulting three-activity trajectory, built around clock-hand movements, tangram arrangements, and parallel-road intersections, was workable in classroom practice and was associated with improvements in both quantitative and qualitative findings. In Cycle 2, the proportion of students in the high category increased from 9% to 45%, while the proportion in the low category decreased from 61% to 18%. Classroom observations and interviews further indicated a shift from visual guessing and isolated computation towards more explicit use of angle definitions and relationships in students' explanations.

Beyond improvements in test scores, the study demonstrates how design research and

HLT refinement can systematically align instructional design with students' actual thinking. The integration of contextual PBL tasks and dynamic GeoGebra exploration supported students in connecting visual representations with formal geometric properties. Taken together, these findings indicate that an effective PBL and GeoGebra design should function as an integrated system in which contextual problems initiate inquiry, dynamic visualisation enables exploration and verification, and structured teacher scaffolding ensures that empirical observations are progressively transformed into mathematical justifications. Although the findings are specific to one school and two design cycles, the refined trajectory provides a theoretically grounded and practically applicable model for teaching lines and angles in Grade VII classrooms and may serve as a basis for further refinement in other educational contexts.

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