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Adaptive fraction instruction: Design research of numeracy trajectories for blind and deaf students

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ABSTRACT

This study employed Educational Design Research (EDR) to investigate the numeracy literacy profiles of six deaf and two blind junior high school students following the implementation of a tailored curriculum on fraction addition and multiplication. The research comprised three phases: preliminary research, teaching experiments, and retrospective analysis. Observations and interviews were conducted to identify the Actual Learning Trajectory (ALT), which was then compared with the Hypothetical Learning Trajectory (HLT), while tests were administered to assess students' numeracy literacy outcomes. The findings revealed discrepancies between the actual and predicted learning trajectories. For blind students, the Audiobook–Braille method supported the development of two of the three numeracy literacy indicators: using numbers and mathematical symbols, and interpreting results. However, students continued to experience difficulties in analyzing information presented in various forms due to the limitations of Braille in representing symbolic information. For deaf students, augmented reality (AR) and visual scaffolding facilitated initial success; however, students struggled with vertical mathematization, particularly in transitioning from concrete models to abstract procedures, indicating a strong reliance on visual representations. The study was limited by the modality-specific constraints experienced by each group. Future research should develop tactile media to support blind students' understanding of the least common multiple (LCM). Further investigations should also examine the effects of time constraints on deaf students' numeracy literacy outcomes and develop curricula that explicitly support the transition from *model of* to *model for* to strengthen vertical mathematization.

KEYWORDS

Inclusive education; numeracy literacy; fraction learning; blind learners; deaf learners

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INTRODUCTION

Real-time information processing has become a fundamental necessity in everyday life (Bozkurt et al., 2024; Jia et al., 2024). This ability is closely related to numeracy literacy, which refers to an individual's proficiency in mathematical reasoning, including the ability to formulate, employ, and interpret mathematical concepts to solve problems. Developing numeracy literacy is therefore considered essential (Wannenburg & Curlewis, 2023). One of the most challenging yet crucial topics in mathematics is fractions. A solid understanding of fraction concepts is critical for success in advanced mathematics, while deficits in fraction understanding have been associated with difficulties in occupational achievement and financial decision-making in adulthood (Mousley & Kurz, 2015; Fan et al., 2025). Fractions are inherently challenging because they require an understanding of magnitude that

contradicts previously learned rules about whole numbers (Fan et al., 2025). Common difficulties experienced by students include conceptual understanding, the execution of operational procedures, and the application of fraction concepts to everyday problem-solving (Diniarti et al., 2023; Sari et al., 2024).

For children with special needs (CSN), these challenges are further exacerbated by sensory barriers that hinder their understanding of fraction magnitudes and symbolic representations. To address these difficulties, mathematics instruction aimed at developing numeracy literacy should involve explicit instruction, systematic prompting, smaller instructional chunks, frequent repetition, and the breakdown of complex problems into manageable steps (Sankalaite et al., 2023). Preliminary observations and diagnostic assessments revealed that blind and deaf students experienced significant difficulties with fractions. Specifically, teachers reported that although students could perform basic whole-number arithmetic, they struggled to understand fraction parts (e.g., 1/2 versus 1/4) and interpret fraction-related information in simple practical contexts. These preliminary findings suggest that existing instructional approaches fail to bridge the gap between abstract fraction symbols and the sensory modalities through which these students access information.

Over the past decade, instructional design research has attracted increasing attention from researchers. A systematic literature review of 163 publications from 2013 to 2020 demonstrated the growing role of design research in addressing the need to improve the quality of classroom instruction (Tinoca et al., 2022). Instructional designs incorporating appropriate learning activities and stimuli can enhance students' mathematical understanding (Srikongchan et al., 2021; Tong et al., 2021). Developing effective instructional designs for fractions is equally important in mainstream classrooms and those serving children with special needs (CSN) (Pires et al., 2022). In this study, adaptive teaching is designed to address fraction-related numeracy barriers by drawing on the principles of Universal Design for Learning (UDL) (Sharma, 2024). This approach emphasizes adapting instructional modalities by translating visual representations of fractions into tactile-auditory formats for blind students and linguistic-visual formats supported by augmented reality (AR) for deaf students. These adaptations are intended to facilitate the transition from real-world objects to abstract mathematical representations (Rahmawati & Pandjaitan, 2020).

The diverse characteristics and learning needs of individual CSN students necessitate specific interventions to maximize their potential and minimize learning barriers. Special education requires the adaptation of instructional media and methods to accommodate

students' individual learning needs (Maryanti et al., 2021; Nurmayani et al., 2025). Adaptive teaching is grounded in the principles of Universal Design for Learning, which emphasize designing flexible learning environments and adapting instructional approaches to accommodate learner diversity (Sharma, 2024). This approach incorporates multisensory strategies (visual, auditory, and kinesthetic) to reinforce learning and often begins with real-world objects or three-dimensional (3D) materials before transitioning to symbolic representations (Rahmawati & Pandjaitan, 2020). The effectiveness of adaptive teaching for CSN depends on strategically selecting and adapting instructional modalities to accommodate the specific sensory needs of individual students.

For blind students, the primary challenge is limited access to visual and geometric representations, which dominate mathematics instruction and hinder their understanding of spatial relationships (Sukarma et al., 2024). Accordingly, instructional adaptations for blind students focus on compensating for limited visual access by maximizing tactile and auditory modalities (Maćkowski et al., 2023; Riga & Kouroupetroglou, 2024). Such adaptations may involve tactile learning media, including Braille, three-dimensional manipulatives (e.g., wooden fraction segments or sliced fruit), Numicon, and tactile graphs, which require explicit instruction to ensure accurate interpretation (Riga & Kouroupetroglou, 2024; Zeinullin, 2025). Additional support can be provided through assistive technologies, such as screen readers, Braille displays, and specialized applications. In contrast, instructional adaptations for deaf students focus on leveraging their visual strengths while addressing linguistic barriers (Mohamad & Sudana, 2024). Deaf students often experience difficulties understanding complex mathematical terminology and linguistic structures, such as conditional and comparative statements, in word problems, which may increase working memory demands (Bashir et al., 2024). Consequently, adaptive teaching for deaf students requires teachers to possess strong sign language proficiency, facilitate meaningful discussions about fractions, and emphasize clear and consistent visual representations (e.g., diagrams) to reduce conceptual confusion.

The fundamental differences between these adaptive approaches—tactile-auditory adaptations for blind students and linguistic-visual adaptations for deaf students—raise critical questions about their respective effects on learning outcomes. Evaluating these effects requires a comprehensive assessment framework. Numeracy profiles capture students' existing knowledge, skills, and learning needs across three main components: Content (core mathematical knowledge, including understanding fraction magnitude), Context (the

application of mathematics to real-world situations, such as those encountered in the workplace or community), and Cognitive and Affective Processes (including reasoning, working memory, and inhibitory control) (Cokely et al., 2018). In this study, numeracy literacy is operationalized through three indicators: (a) using numbers and mathematical symbols related to basic mathematics to solve practical problems in a wide range of everyday contexts; (b) analyzing information presented in various forms, such as graphs, tables, and charts; and (c) interpreting analytical results to make predictions and decisions (Han et al., 2017). The numeracy profiles also document potential barriers, including linguistic and sensory constraints, to contextualize students' performance.

Although blind and deaf students are often grouped under the category of sensory impairments, the literature indicates differences in the learning environments associated with their academic performance. Deaf students have been reported to perform better in special or segregated schools, potentially because of greater access to instruction adapted to their linguistic needs, particularly sign language. In contrast, blind students have been reported to perform better in mainstream or inclusive educational settings (Rosas et al., 2023). These differences underscore the need for comparative investigations. However, the literature remains limited in studies that systematically examine and compare numeracy profiles following interventions involving two fundamentally different adaptive instructional modalities—tactile-auditory and linguistic-visual—in fraction instruction. Therefore, this study aims to address this gap by analyzing and comparing the numeracy profiles of blind and deaf students following adaptive fraction instruction, thereby contributing empirical evidence to inform the development of instructional approaches for CSN.

METHODS

This study employed Design Research (DR) of the validation study type. Validation studies focus on developing, testing, and validating instructional theories (Van den Akker et al., 2006). DR consists of three phases: preliminary research, teaching experiments (the prototyping phase), and retrospective analysis. This study adopted a predominantly qualitative approach, integrating quantitative outcomes from numeracy tests into a qualitative analytical framework. Quantitative test scores were not used for statistical inference or generalization; rather, they served as diagnostic anchors for constructing descriptive qualitative profiles and analyzing specific response patterns in relation to students' Actual Learning Trajectories (ALTs) and the Hypothetical Learning Trajectory (HLT). This article specifically focuses on the evaluation of the implemented instructional design.

Research Subjects

The study was conducted in special education settings involving students with visual impairments (blind students) and hearing impairments (deaf students). The primary objective was to investigate and describe students' numeracy literacy profiles following the implementation of the instructional design by analyzing the outcomes of the teaching experiment phase.

Participants were selected using purposive sampling from two special education schools: a Type A special school for blind students and a Type B special school for deaf students. The rationale for selecting these participants was based on the distinct sensory modalities through which the two groups access learning information (tactile-auditory vs. linguistic-visual), providing an opportunity to examine modality-based instructional adaptations in fraction learning. The participants were junior high school students in Grades 7 and 8, aged 13–16 years, and were categorized into two groups, as presented in Table 1.

Table 1. Research Subjects

Research Phase	Disability Group	School	Grade Level	Number of Participants (*n*)	Student Initials
Pilot Study	Blind Students (Visual Impairment)	SLB A	VIII	3	HAS, RK, RVA
	Deaf Students (Hearing Impairment)	SLB-B	VII	3	MI, AJ, R
Main Teaching Experiment	Blind Students (Visual Impairment)	SLB A	VII	2	ASP, SL
	Deaf Students (Hearing Impairment)	SLB-B	VII	6	TO, DK, ZH, RD, DK, PS

Data Collection Instruments

Data on students' numeracy literacy competencies were collected using a numeracy literacy test. The primary instrument was a specialized Fraction Numeracy Literacy Test designed to assess three components of numeracy literacy (Han et al., 2017): (a) using numbers and mathematical symbols in everyday contexts, (b) analyzing mathematical information presented in various formats, and (c) interpreting results to support decision-making. The test items assessed students' abilities to understand, apply, and communicate mathematical concepts in various real-life contexts.

The test was specifically developed and validated to align with the curriculum objectives while accommodating the learning needs of students with visual and hearing

impairments. The mathematical content remained identical across groups, but the presentation formats were adapted to ensure accessibility for each group. For blind students, textual items were transcribed into standard Grade II Braille, while visual diagrams were either omitted or converted into descriptive auditory cues delivered through audiobooks. For deaf students, linguistic structures were simplified to reduce syntax-related working memory demands, particularly by avoiding complex conditional clauses. Abstract word problems were also supplemented with visual markers presented through augmented reality (AR).

Instrument Validity

The Hypothetical Learning Trajectory (HLT), student activity sheets, initial ability baseline tests, and final numeracy literacy assessment instruments underwent comprehensive evaluation by a multidisciplinary panel of experts. The panel comprised five specialists: two mathematics education lecturers, one expert in special education needs (SEN) pedagogy, one specialist teacher for blind students, and one specialist teacher for deaf students.

Rather than relying solely on conventional validation procedures that focus on assigning a quantitative validity classification, the expert review in this study adopted a developmental approach consistent with the design research framework (Van den Akker et al., 2006; McKenney & Reeves, 2019). The primary objective was to obtain qualitative feedback, identify necessary revisions, and facilitate constructive discussions to progressively refine the instructional design prototypes through multiple iterations before their implementation in the classroom. The involvement of multidisciplinary experts helped ensure that the resulting instructional design was both theoretically grounded and practically applicable in special education settings (Plomp & Nieveen, 2013). Collaboration between mathematics educators and special education practitioners also supported the effective and accessible integration of digital tools into inclusive mathematics instruction (Schoenfeld, 2014).

Data Validity and Reliability

To ensure methodological rigor, this study addressed qualitative validity and reliability to establish the credibility and dependability of the findings within special education contexts (Gravemeijer & Cobb, 2006; McKenney & Reeves, 2019). Unlike psychometric approaches to instrument validation, qualitative validity in this study emphasized the accuracy and consistency of data collection and interpretation. The Hypothetical Learning Trajectory (HLT) served as a central analytical reference for supporting construct validity and ensuring that the analytical conclusions could be traced to the collected evidence (Bakker, 2018).

The HLT also supported interpretive validity by enabling researchers to systematically examine whether students achieved the intended learning milestones and to identify learning developments that might extend beyond initial expectations (Drijvers et al., 2021). Meanwhile, qualitative reliability referred to the trustworthiness and dependability of the research findings under comparable instructional conditions (Plomp & Nieveen, 2013).

Rather than relying on statistical formulas or coefficients, reliability was established through two qualitative procedures. The first was data triangulation, which involved corroborating findings across multiple sources of evidence, including observational notes, interview transcripts, and student-produced artifacts (Creswell & Poth, 2018). The second was cross-interpretation, in which multiple researchers independently analyzed the same dataset to minimize individual researcher bias and enhance the objectivity of the interpretation (Schoenfeld, 2014). Combining these procedures with evidence from multiple data sources strengthened the credibility and dependability of the qualitative findings (Schoenfeld, 2014; Cobb et al., 2017).

Data Analysis

This study employed a predominantly qualitative approach to generate detailed interpretative accounts and develop a comprehensive understanding of students' numeracy literacy competencies within their specific learning contexts (Thanh & Thanh, 2015). The collected data, including test responses, observations, and interview findings, were analyzed using the interactive model proposed by Miles et al. (2019). The analysis comprised three interconnected stages:

1. Data Reduction: This stage involved selecting, focusing, abstracting, and transforming raw data, including test scores, student responses, and observation notes, into meaningful summaries. The process aimed to identify emerging patterns and categorize students' responses according to the different indicators of numeracy literacy.
2. Data Display: The reduced data were organized and presented in matrices, tables, and narrative descriptions. This stage facilitated a systematic and comprehensive examination of students' numeracy literacy profiles across different test items and the two participant groups (blind and deaf students).
3. Conclusion Drawing and Verification: Conclusions were drawn by interpreting the displayed data, with particular attention to identifying and verifying emerging patterns and themes related to students' numeracy literacy profiles. Verification involved cross-checking the findings against the instructional design objectives to ensure that the

conclusions were adequately supported by the collected evidence.

RESULT AND DISCUSSION

Instructional Design

This study employed a design research approach to develop specialized and adaptive instructional designs for students with special educational needs (SEN), specifically blind and deaf students, focusing on the topic of fractions. These specialized instructional designs are conceptualized as Local Instructional Theories (LITs). The primary distinction between the two designs lies in their use of the dominant sensory modalities of each student group, which serve as the foundation for formulating the instructional hypotheses, namely the Hypothetical Learning Trajectories (HLTs). Table 2 presents a comparison of the instructional designs developed for blind and deaf students.

Table 2. Comparison of Instructional Design for Blind and Deaf Students

Aspect	Blind students	Deaf students
Primary Input Modality	Auditory (Audiobook), Tactile (Feeling)	Visual (AR), Kinesthetic (Origami/Shading Bar), Sign Language
LIT Context	Maksuba Cake (Tactile Tool)	Table/Window Glass (AR Visualization)
Critical Activities	Comparing Fractions, LCM Concept (Geo-board)	Equivalent Fractions (AR), Unlike Denominator Addition (Origami/Shading Bar)
Main Conceptual Barrier	Non-Visual Abstraction (LCM), Braille Expression Limitation	Visual Model Dependence (AR/Origami), Vertical Mathematization

For blind students, who rely primarily on auditory and tactile input, the instructional scaffolding focused on integrating audiobooks with Braille materials. Audiobooks provided contextual information (textual content) for word problems, such as the *Maksuba* cake context, while Fraction Blocks and Geo-boards (*Papan Paku*) served as concrete manipulative materials. This instructional design aimed to develop students' conceptual understanding of fractions through tactile and auditory experiences, thereby addressing the challenges associated with visual representation and mental imagery.

In contrast, for deaf students, the instructional design emphasized visual and kinesthetic approaches to maximize their access to mathematical information. Augmented reality (AR) provided rich and dynamic contextual representations, often supplemented by sign language (BI) videos to support linguistic comprehension. Kinesthetic materials, such as origami and shading bars, were incorporated to facilitate mathematical modeling and the spatial representation of fraction concepts. This design aimed to facilitate the transition from abstract concepts to meaningful understanding through visual and physical interactions,

taking into account the students' learning characteristics.

Numeracy Literacy

Three main indicators were used to assess students' numeracy literacy (ILN) following the implementation of the instructional design: the ability to use numbers and mathematical symbols (ILN 1), the ability to analyze information (ILN 2), and the ability to interpret results for decision-making (ILN 3) (Han et al., 2017).

Indicator 1 (ILN 1): Using Mathematical Symbols and Basic Operations in Real-life Context

ILN 1 refers to an individual's ability to use various numbers and mathematical symbols related to basic mathematics to solve everyday problems. Analysis of the Final Test (TA) results during the teaching experiment revealed a difference in achievement between the two groups. Deaf students demonstrated high levels of achievement, with 100% (6 out of 6 students) successfully meeting ILN 1. This finding suggests that visual scaffolding, particularly AR, effectively supported the *Employ* stage, which involves translating contextual problems into fractional notation and applying appropriate mathematical procedures.

In contrast, blind students demonstrated inconsistent achievement. Only 50% (1 out of 2 students, ASP) achieved satisfactory results on ILN 1 in the Teaching Experiment Final Test (TA), while the other student (SL) did not meet the indicator. Further analysis revealed that SL's difficulties stemmed partly from limited proficiency in Braille mathematical notation, which is essential for reading, writing, and understanding basic fraction operations. As a student who had recently transferred from a mainstream school and had not yet developed sufficient Braille proficiency, SL experienced considerable difficulties in expressing mathematical symbols, which became a major barrier to performing the operational procedures required to meet ILN 1.

Indicator 2 (ILN 2): Analyzing Information Presented in Various Representations

ILN 2 assesses students' ability to analyze information presented in various forms, such as tables, graphs, and diagrams, to solve contextual problems. Deaf students demonstrated stronger performance on this indicator, with 100% (6 out of 6 students) successfully meeting ILN 2 in the Teaching Experiment Final Test (TA). This achievement was supported by instructional activities that encouraged students to interpret visual and tabular information, particularly Activity 3, which incorporated tables and shading bars.

For blind students, ILN 2 emerged as a significant area of conceptual difficulty. Although one student (ASP) demonstrated the ability to analyze a diagram in a specific test

item (TA Q3), the students generally experienced difficulties independently formulating the least common multiple (LCM) procedure during Activities 3 and 4. These difficulties were associated with transferring an abstract, non-visual understanding of LCM into systematic procedures for determining common denominators. Understanding contextual information through auditory input alone did not necessarily translate into the ability to perform procedural analysis without adequate tactile representations to support abstract mathematical operations.

Indicator 3 (ILN 3): Interpreting Results and Making Decisions

ILN 3 represents the highest level of numeracy literacy assessment, focusing on students' ability to interpret and communicate analytical findings to make predictions and draw conclusions. Achievement on this indicator was relatively similar across the two groups, although the overall success rate remained moderate. Deaf students achieved a 50% success rate (3 out of 6 students) in meeting ILN 3 in the Teaching Experiment Final Test (TA). Difficulties experienced by the remaining three students were frequently associated with strict time constraints, which prevented them from completing complex mathematical operations required for ILN 2 and subsequently drawing valid conclusions within the allotted time.

Blind students also demonstrated a 50% achievement rate (1 out of 2 students) on ILN 3. However, this achievement was affected by procedural difficulties encountered during ILN 2. Errors in mathematical procedures, such as determining equivalent fractions, directly affected the accuracy and validity of students' final conclusions. These findings suggest that ILN 3, which involves interpreting analytical results, is highly dependent on the accuracy of the preceding analytical stage (ILN 2). For blind students, these difficulties were further compounded by the additional time required to read and write mathematical symbols in Braille.

Challenges Experienced by Blind Students

Understanding abstract fraction concepts, particularly addition involving fractions with unlike denominators, presented a major challenge for blind students. Although audiobooks successfully facilitated contextual understanding through auditory input, translating tactile representations, such as Fraction Blocks and Geo-boards, into the abstract procedure for determining the LCM required a substantial cognitive transition. The absence of visual representations meant that students had to rely heavily on tactile exploration and mental processing to identify common multiples, potentially increasing working memory

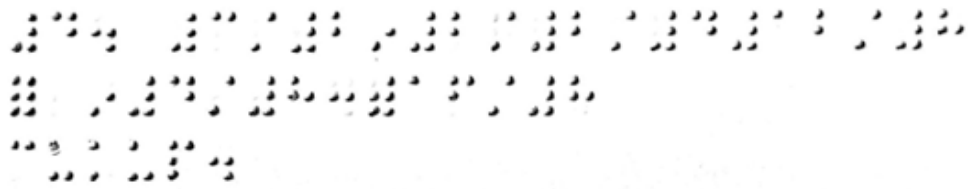
Kalamatika: Jurnal Pendidikan Matematika
Volume 11, No. 2, November 2026, pages 251-271
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demands.

This difficulty created challenges in vertical mathematization. Although students could explore concrete models through touch and follow mathematical instructions through auditory input, they struggled to generalize these experiences into formal mathematical procedures. Empirical evidence of this difficulty can be observed in student HAS's initial error when solving a basic fraction addition problem, in which the student calculated 3/10 + 5/10 = 30/10. This response illustrates weaknesses in foundational fraction arithmetic that needed to be addressed before introducing more complex concepts, such as finding the LCM.

The challenges experienced by blind students were not limited to conceptual understanding but also involved difficulties in symbolic mathematical expression. Limited proficiency in Braille mathematical notation, particularly among transfer students such as SL, significantly affected their achievement in ILN 1 and ILN 3. When students require additional time to recognize, interpret, or write mathematical symbols in Braille, their cognitive processing demands increase.

Consequently, the already demanding abstract reasoning process becomes further complicated by the additional time required to translate mathematical information into Braille notation. This may affect calculation accuracy and reduce the time available for completing the interpretation stage (ILN 3), thereby contributing to difficulties in solving more complex problems in the Final Test (TA). The following examples of student responses illustrate these challenges in greater detail.



The response provided by the blind student in Activity 4 of the Teaching Experiment (fraction addition) provides concrete evidence of procedural difficulties. When solving the word problem involving 3/2 + 2/4, the student demonstrated the following calculation steps: 3/2 + 2/4 = 12/8 + 4/8 = 16/8.

Figure 1. Student Evidence, Case 1: Blind student

The analysis (see Figure 1) shows that the student successfully recognized the need to find a common denominator and identified 4 as the least common multiple (LCM). However, an error occurred in the subsequent conversion to equivalent fractions. The student converted (3/2) into (12/8), rather than using the denominator 4 as intended. This suggests that although the concept of the LCM had been introduced, the student's ability to apply it accurately when

converting fractions remained underdeveloped (ILN 2). This difficulty may have been compounded by the slow and error-prone process of writing mathematical symbols in Braille.

The visually rich LIT provided advantages in supporting ILN 1 and ILN 2 among deaf students, with augmented reality (AR) and origami functioning as *models of* the problem—that is, as concrete representations of specific mathematical situations. However, this success was accompanied by a strong dependence on visual representations.

A fundamental difficulty identified in the teaching experiment was the students' limited progress in vertical mathematization, particularly in transitioning from a concrete visual model (e.g., folding origami to represent (1/3 + 1/4 = 7/12)) to the formulation of a systematic abstract procedure (*model for*), such as the general rule for finding the LCM. Although students could visually recognize equivalent fractions (e.g., (1/2) and (2/4)), they struggled to explain and formulate the abstract reasoning behind multiplying (1/3) by (4/4) to obtain an equivalent fraction with a common denominator. Empirical evidence from Student MI during Activity 1 of the Pilot Experiment (fraction addition) showed that the student could visually represent (1/2) and (2/4) but needed guidance to formally establish that the two fractions were equivalent. This suggests that visual models can support mathematical representation, but do not necessarily lead to the automatic development of formal conceptual understanding.

A common misconception, such as adding the numerators and denominators separately, may arise from incomplete vertical mathematization. Students may apply operational rules without understanding the principle that fractions can be added directly only when they represent parts of the same-sized fractional unit—that is, when they have a common denominator.

This constraint may be further compounded by communication-related challenges. Although Indonesian Sign Language (BISINDO) and the Indonesian Sign System (SIBI) can support the communication of contextual narratives, expressing abstract mathematical concepts and specialized mathematical vocabulary through sign language may present additional challenges. This can make it more difficult for teachers to provide formal and systematic explanations of the principles underlying vertical mathematization. Figure 2 provides an example of a student's written response.

Kalamatika: Jurnal Pendidikan Matematika
Volume 11, No. 2, November 2026, pages 251-271
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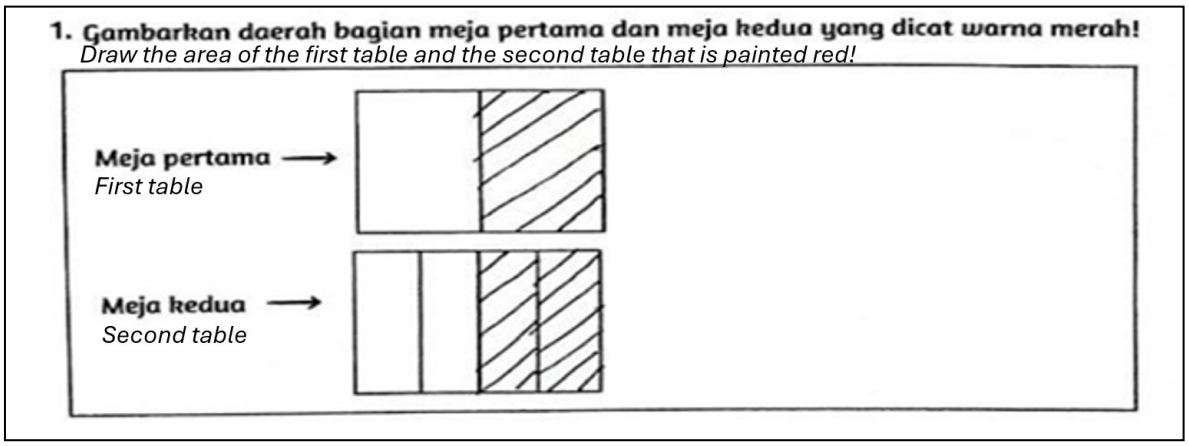


Figure 2. Student Evidence, Case 2: deaf student

The response of Student MI in Activity 1 of the Pilot Experiment (fraction addition) focused on modeling equivalent fractions (1/2 and 2/4) in a table context. After observing the AR visualization, the student successfully represented the shaded regions. However, the student initially required direct guidance from the researcher to formally conclude that the two fractions were equivalent. This finding reinforces the observation that AR-based visual scaffolding successfully supported the construction of a detailed *model of* the mathematical situation (ILN 1). However, the visual representation did not automatically lead to the formal understanding of equivalent fractions. Linguistic intervention and probing questions were still needed to help the student move from visualization to conceptual abstraction, indicating the need for additional scaffolding in developing ILN 2.

Modality-Based Evaluation of Student Numeracy Literacy Profiles

The numeracy literacy profiles indicate that deaf students, supported by visual adaptations through Augmented Reality (AR), demonstrated higher and more consistent achievement in the initial stages of numeracy literacy—namely, the use of numbers and mathematical symbols (ILN 1) and the analysis of information (ILN 2)—than blind students in the context of fraction addition. All deaf students (100%) achieved ILN 1 and ILN 2 during the teaching experiment. These findings are consistent with previous research suggesting that strategies such as sign language interpretation, interactive learning activities, and visual aids can support the mathematical understanding and achievement of deaf students (Bashir et al., 2024).

For blind students, the main challenges involved non-visual procedural abstraction, particularly in understanding and applying the least common multiple (LCM), and symbolic literacy in the Braille mathematical code (KMT). For deaf students, the primary challenge was their strong dependence on visual models, which appeared to hinder the process of

vertical mathematization—the transition from an AR-based *model of* a mathematical situation to an abstract *model for* mathematical reasoning. In both groups, performance in interpreting results and making predictions or decisions (ILN 3) was affected by different constraints, including procedural difficulties and external factors such as limited instructional time. Based on these findings, further research should focus on developing tactile instructional media that explicitly support blind students in abstracting and applying the LCM concept. In addition, further investigation is needed into the effects of strict time constraints on deaf students' performance in ILN 3. Future studies should also explore instructional approaches that explicitly support the transition from *model of* to *model for* to strengthen vertical mathematization among deaf students.

The empirical profiles generated in this study both corroborate and extend existing literature on adaptive mathematics instruction for learners with sensory disabilities. The achievement of deaf students in symbolic representation (ILN 1) and the analysis of visual information (ILN 2) highlights the importance of visual and multisensory instructional scaffolding. Rahmawati and Pandjaitan (2020) argue that such scaffolds, including augmented reality (AR), can help address linguistic barriers in the education of deaf and hard-of-hearing students. However, the present study identifies an important limitation: strict time constraints may hinder deaf students from completing higher-order interpretative tasks (ILN 3). This finding adds a relevant consideration to research on adaptive instruction for students with a single sensory disability. Conversely, the 50% rate of difficulty among blind students in basic mathematical notation (ILN 1) is consistent with Sharma's (2024) discussion of the cognitive demands associated with translating mathematical information into tactile Braille representations. The cross-modality comparison in this study further suggests that difficulties in reading and writing mathematical Braille can become a bottleneck, potentially affecting students' procedural performance in LCM abstraction (ILN 2) and subsequent interpretative reasoning (ILN 3).

Theoretically, these findings have implications for implementing Realistic Mathematics Education (RME) and Universal Design for Learning (UDL) in special education. The study highlights that a Hypothetical Learning Trajectory (HLT) should not be understood solely as a sequence of cognitive milestones; its implementation must also account for the physical and linguistic modalities through which learners access and process mathematical information (Bakker, 2018; Van den Akker, 2006). Tactile and visual learning trajectories may differ considerably in the transition from a concrete *model of* a mathematical

situation to a more abstract *model for* mathematical reasoning (Drijvers et al., 2021). The development of higher-order interpretative skills (ILN 3) may also depend on the processing demands imposed by the instructional medium. For blind students, the time and cognitive effort required to read and write mathematical Braille may place additional demands on working memory. For deaf students, excessive reliance on visual AR models may limit the development of more abstract mathematical representations. These findings support a shift from general adaptive instruction toward a modality-specific scaffolding framework in which instructional designs explicitly establish connections between concrete sensory representations and abstract mathematical notation (Schoenfeld, 2014; Cobb et al., 2017).

CONCLUSION

The evaluation of the instructional design provided profiles of the numeracy literacy competencies of blind and deaf students across three indicators: using numbers and mathematical symbols, analyzing information, and interpreting results to make predictions and draw conclusions. This study contributes to the field of special education by presenting a comparative analysis of how different modality-based instructional approaches—Audiobook–Braille and Augmented Reality (AR)—relate to the Actual Learning Trajectories (ALTs) of two groups of students with different sensory disabilities within the same mathematical topic. Unlike studies that examine adaptive mathematics instruction for blind and deaf students separately, this study compares the challenges each group encounters when moving from concrete instructional media toward abstract mathematical representations.

The findings indicate that the Audiobook–Braille approach supported the development of foundational numeracy literacy skills among blind students. However, challenges remained in the second and third indicators, particularly those associated with Braille mathematical literacy and the abstraction of the least common multiple (LCM). For deaf students, AR and visual scaffolding supported achievement in ILN 1 and ILN 2 among all participants. However, only half of the students demonstrated achievement in ILN 3. Their learning trajectory was also characterized by a strong reliance on visual models and difficulties with vertical mathematization, particularly when moving from a *model of* a mathematical situation to a *model for* abstract reasoning. Limited instructional time further constrained their performance in higher-order interpretative tasks.

To address these limitations, future studies should focus on developing modality-specific tactile instructional media, such as embossed fraction grids, physical fraction tiles with Braille overlays, and dynamic thermoform graphics. These materials should explicitly

connect concrete representations of fractional magnitudes with abstract LCM procedures and symbolic representations for blind students. For deaf students, further investigations should focus on strengthening the third numeracy literacy indicator—interpreting results to make predictions and draw conclusions—through instructional scaffolding that gradually reduces dependence on visual AR representations and supports the development of formal mathematical reasoning. Finally, instead of relying exclusively on short-term teaching experiments, future research could employ a longitudinal design spanning at least two academic semesters to examine the sustained effects, retention, and transferability of these modality-specific instructional adaptations across other mathematical topics, such as geometry and algebraic thinking.

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