THE NEWMAN PROCEDURE FOR ANALYZING STUDENTS’ ERRORS IN SOLVING SYSTEMS OF LINEAR EQUATIONS

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ABSTRACT

This study aimed to analyze students’ errors in problem-solving activities for systems of linear equations. The descriptive qualitative method was adopted and applied to obtain and process the research data. Furthermore, research subjects were selected using the purposive sampling technique. Three participants were chosen according to their mathematical proficiency levels. Data collection was conducted by tests to measure students’ problem-solving abilities and semi-structural interviews to gather qualitative information about students’ errors in solving systems of linear equations. The interview results were analyzed using narrative analysis to obtain accurate conclusions about students’ errors. Furthermore, the study found that (1) low-ability students tend to perform error at the comprehension stage, (2) medium-ability students are likely to perform error at the transformation stage, and (3) high-ability students tend to perform error at the process skills stage. The solutions based on the ability level are: (1) low-ability students are required to read the question carefully, educators should emphasize the problem-solving procedure, and students should strengthen their understanding of the prerequisite learning content in problem-solving; (2) medium-ability students have to focus on the emphasis and development of their skills in understanding the language of a problem and balance with improving their understanding of learning content and contextual exercising; and (3) high-ability students are provided with exercises that can improve their counting speed and accuracy of the subject in resolving a problem.

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INTRODUCTION

Mathematics learning is inseparable from mathematics problems. Each educator has a different teaching strategy for mathematics problem-solving. While problem-solving is the main challenge for students in mathematics learning (Abidin, 2015). A problem-solving will be difficult if the student does not have the algorithm or procedure to solve it (Bauer, Flatten, & Popović, 2017; Serin, 2020). Students have to use prior knowledge in solving problems to gain the new one (Hasbullah & Wibawa, 2017). This statement means that students must connect their previous knowledge with the problems and develop their completion. Problem-solving ability is one of the must-have mathematical skills for students at school (Akbar et al., 2017; Ramdan et al., 2018; Sumartini, 2016; Ulandari et al., 2019). Therefore, it requires further students’ errors analysis in solving problems.

Students' problem analysis could solve a challenge in the teaching and learning process (Verschaffel, Greer, & De Corte, 2010). This error analysis is expected to identify the cause of the student's error and be a basis to overcome the learning obstacles. An error analysis procedure that could be used is Newman error analysis. A teacher developed Newman's error analysis in Australia in 1977. Newman defines five crucial activities in analyzing the error of solving a problem: reading, understanding, transformation, process skills, and encoding (White, 2010). Newman's procedure was used because it has easy and straightforward steps to identify students' errors and difficulties in problem-solving.

Students' errors in mathematics problem-solving could vary. One of the factors is the basic mathematics understanding of each individual. Thus, the error analysis divides students into groups or categories that correspond to students' mathematics ability. The error descriptions could be reviewed from students’ initial mathematical ability: high, medium, and low. Afterward, it could also be described as the cause of errors based on each group's error. This statement corresponds to Ruseffendi (2006), which revealed that techniques for analyzing assessment results could be classified according to students’ proficiency level, level of understanding, and individual expertise.

Based on the previous explanation, error analysis and solutions to students’ mathematics problem-solving challenge is required to achieve learning objectives.
METHOD

This study utilized a qualitative approach with a case study method to depict and describe students' errors in problem-solving activities for a linear equation system topic. Sugiono (2012) suggested that the characteristic of qualitative descriptive research is to examine the original condition or the actual situation. A case study is a research strategy whereby researchers investigate an event, activity, process, or group of individuals limited by time and activity using various data collection procedures based on time specified (Creswell, 2014). Case studies could present in-depth and detailed perspectives of students' errors in problem-solving activities. Thus, the case study is appropriate to the purpose of this research.

The samples of this study were 15 students of the Mathematics Education Program in College of Teacher Training and Education (STKIP) MNC who were selected purposively. The participants were selected based on their linear algebra examination scores. Next, one subject was chosen to represent each group of the low-ability, medium-ability, and high-ability. In this study, the data collection was conducted through a test to measure students’ problem-solving abilities and a semi-structural interview to gather beneficial qualitative information related to students' errors in solving the linear equation system problem. The interview data were analyzed using narrative analysis to obtain accurate information about students' errors (Cohen, Manion, & Morrison, 2018).

RESULTS AND DISCUSSION

The study results are based on the test and interviews with research participants on the problem-solving process in a linear equation system. The problems designed for this study is as follows:

A chemist mixes three glucose solutions with a concentration of 20%, 30%, and 45% to produce 10 L of a glucose solution at a concentration of 38%. If the 30% solution volume used is 1 L more than twice the 20% solution used, specify each solution's volume.

The following are the results of the error analysis towards the three levels of students' mathematical ability.

The subject in the low-ability category (S1)

The result of the subject's work might indicate the error of S1. S1’s answer is presented in Figure 1.
Based on Figure 1, the student did not understand what was meant or required in question, resulting in the subject's failure to solve the problem. Further, the results of the subject’s answer were verified with interviews based on Newman’s procedure.

R: “Please re-read this question!”
S1: “A chemist mixes three glucose solutions with a concentration of 20%, 30%, and 45% to produce 10 L of a glucose solution at a concentration of 38%. If the 30% solution volume used is 1 L more than twice the 20% solution used, specify each solution's volume.”

R: “Do you know the problem that should be solved?”
S1: “Calculate the volume of each solution used. (Only repeated questions in the problem)”

R: “Do you understand the problem?”
S1: “I could not understand the problem, so I could not write the mathematical model.”

R: “What formula should you use to solve the problem?”
S1: “I do not know, I do not understand”.

R: “Please, try to explain how you solved the problem!”
S1: “I do not understand the problem, so I could not develop a solution in calculations. I wrote the answer by estimating the volume number of each solution”.

R: “Why didn't you accurately calculate the calculation process”s.
S1: “Because I could not understand the problem, I was confused”.

R: “Do you understand the learning content of the linear equations system well?”
S1: “Just a little. It is just that I sometimes have a lack of understanding about the word problem”.

R: “What makes you difficult about the word problems?”
S1: “It is difficult to understand the problem and transform it into a mathematical form”.

The interview excerpt illustrates that the subject could not solve the linear equation system's problem well. The participant could not understand the context of the problem, resulting in difficulty developing a problem-solving solution. Furthermore, student’s error analysis results based on Newman’s procedure are presented in Table 1.
Table 1. Error analysis on S1

<table>
<thead>
<tr>
<th>Type of error</th>
<th>Explanation</th>
<th>Interview results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>It could not be investigated through student’s answers.</td>
<td>Read the problem thoroughly and smoothly.</td>
</tr>
<tr>
<td>Comprehension</td>
<td>The subject rewrote the information contained in the question.</td>
<td>The subject could not explain what was known, and the problem was in the mathematical form.</td>
</tr>
<tr>
<td>Transformation</td>
<td>It did not indicate a problem-solving strategy.</td>
<td>The subject had difficulty in creating mathematical models.</td>
</tr>
<tr>
<td>Process skills</td>
<td>• The subject did not perform mathematical procedures. • The subject did not operate all the given information in the problem.</td>
<td>The subject could not understand the problem’s information, so he did not write the calculation process.</td>
</tr>
<tr>
<td>Encoding</td>
<td>• The subject solved the problem without any necessary calculations. • The volume unit was incorrect.</td>
<td>The subject could not explain the written answer.</td>
</tr>
</tbody>
</table>

Table 1 shows that the low-ability subject does not make an error in the reading aspect, but the main error is in the comprehension section. As a result, errors also occur in subsequent stages include transformation, process skills, and encoding. Comprehension error is caused by the subject's inability to understand the information in the problem so that the subject could not create mathematical models and devise a problem-solving strategy. Understanding the problem is essential for students because it can determine the next step to solve the problem (Dogan-Coskun, 2019).

The interview shows that the subject did not correctly implement the problem-solving procedure. The participant had difficulty with word problems. However, this is contrary to pre-existing learning. The subject had been trained with word problems. He recognized that it is difficult to translate the word problem into a mathematical form. Kamsurya (2019) suggested that problems in mathematics learning can be presented in non-routine questions in the form of word problems, depictions of phenomena or events, illustrations of images or puzzles, and the use of social and scientific contexts. Besides, the subject slightly understood the learning content related to the topic. This statement is in line with Rachmawati (2017) and Widyastuti et al. (2017) that low-ability students have difficulty creating mathematical modeling of a problem. Students do not use all given information, confuse in determining the problem-solving strategy, and are unable to perform mathematical procedures. Students' inability to develop problem-solving strategies is a significant problem in mathematics learning nowadays (Thomassen & Stentoft, 2020).

Low-capability students should find the solution to improve their learning outcomes by solving the problem carefully. The subject must also reinforce the prerequisite material understanding in resolving problem-solving. Furthermore, educators should emphasize the
problem-solving procedure to develop an appropriate teaching strategy. In line with this statement, Y. M. Sari & Valentino (2017) argued that the solution for students who do error in understanding the problem is to ask students to read the question carefully and make questions about the problem "what to find or seek a solution" to strategize in problem-solving. Meanwhile, Rachmawati (2017) revealed that alternative solutions for low-ability students are by providing an in-depth understanding of the concept and insight into issues in the form of word problems by carefully teaching them to understand the problem given.

The subject in the medium-ability category (S2)

Subjects in this category could understand the problem and compile what is required in solving the problem, but the subject was unable to design a problem-solving strategy. It is seen in the S2's answer in Figure 2.

![Translation]

**R**: “Please re-read this question!”

**S2**: “A chemist mixes three glucose solutions with a concentration of 20%, 30%, and 45% to produce 10 L of a glucose solution at a concentration of 38%. If the 30% solution volume used is 1 L more than twice the 20% solution used, specify each solution's volume”.

**R**: “Do you know the problem that should be solved”?

**S2**: “Yes, I know that is counting the volume of each solution to create a new solution with a concentration of 38%. So I have to use what is already known in the problem to solve it”.

**R**: “How do you solve the problem?”

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**Figure 2. S2'S Answer**

S2's work was analyzed by the Newman procedure and alignment with the student's interview, as follows.

**R**: “Please re-read this question!”

**S2**: “A chemist mixes three glucose solutions with a concentration of 20%, 30%, and 45% to produce 10 L of a glucose solution at a concentration of 38%. If the 30% solution volume used is 1 L more than twice the 20% solution used, specify each solution's volume”.

**R**: “Do you know the problem that should be solved”?

**S2**: “Yes, I know that is counting the volume of each solution to create a new solution with a concentration of 38%. So I have to use what is already known in the problem to solve it”.

**R**: “How do you solve the problem?”
S2: “First, I have to transform what is known into a mathematical form. However, I was confused about creating the model”.
R: “Why were you confused?”
S2: “I have created two mathematical models. However, when calculating using a linear equation system procedures, it apparently cannot be resumed”.
R: “Why can it not proceed? Are there any missed steps in mathematical modeling?”
S2: “I could not”.

The interview excerpt shows that the S2 subject could understand the linear equation system’s problem well. However, the S2 participant had difficulty creating a mathematical model based on the problems, resulting in an incorrect solution. The analysis of student’s errors based on the Newman procedure is found in Table 2.

<table>
<thead>
<tr>
<th>Type of error</th>
<th>Explanation</th>
<th>Interview results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reading</td>
<td>It cannot be investigated through the subject’s work results.</td>
<td>Read the problem thoroughly and smoothly. The student was explaining what was known and the problem to determine the volume of each solution used.</td>
</tr>
<tr>
<td>Comprehension</td>
<td>Students wrote down known and asked information in the mathematical model but were not yet perfect.</td>
<td>The student had difficulty in creating mathematical models.</td>
</tr>
<tr>
<td>Transformation</td>
<td>• The subject conducted an error in resolving the equations. • The subject did not use the linear equations system strategy properly because there was a shortage in drafting mathematical models.</td>
<td>The subject made an error at the transformation stage, so he could not continue the calculation process. The answers did not convince the subject.</td>
</tr>
<tr>
<td>Process skills</td>
<td>• The subject misused mathematical operations. • The subject did not continue the calculation process.</td>
<td></td>
</tr>
<tr>
<td>Encoding</td>
<td>• The subject wrote the conclusion without going through the correct calculation process. • The subject did not use volume units.</td>
<td></td>
</tr>
</tbody>
</table>

Based on Table 2, S2 had difficulty in creating mathematical models. The medium-ability student often has difficulties in mathematical modeling, determining problem-solving strategies, and applying mathematical procedures (Widyastuti et al., 2017). The main error conducted by S2, according to Table 2, is located in the transformation section. There was a shortage of mathematical models that should be regenerated. However, the subject was unaware of the error, so he succeeded in determining the problem-solving strategy.

The interview results show that the subject could explain the problem and draft the mathematical model. The ability to explain and reasoning that supports mathematical problem-solving is essential for students to solve problems (Rawani, Putri, & Hapizah, 2019). However, the subject showed his doubts in drafting the mathematical model of the question since he could not continue the calculation process using a linear equations system. After being given further questions, the participant was unaware of his error in drafting mathematical models. The interview results confirm that the S2’s error is in the part of the
transformation. The subject could understand the problem but could not compile the mathematical model as the first step to solve the problem. The error continued at the next stage, process skills, and encoding. Zakaria et al. (2010) also explained that the error of process skills always follows the transformation error.

An alternative solution that can be given to a medium-capable subject is to improve the mastery of the learning content covering the concepts and provide students with some exercises related to word problems to understand problem-solving (Rachmawati, 2017) better. Furthermore, according to Zakaria et al. (2010), transformation error happens because the teacher does not emphasize students’ understanding of the mathematical language and skills. Based on this statement, the solution that could be offered to medium-capable subjects is to conduct mathematical learning that focuses on the emphasis and development of students’ abilities in understanding a problem’s language and balanced with improved learning content mastery and contextual exercise (Erdoğan & Gül, 2020).

**Subjects in the high-ability category (S3)**

The S3’s answer indicates that the subject could determine a problem-solving strategy but made an error in his calculation procedure, as seen in Figure 3.

![Figure 3. S3’s Answer](image-url)

Translation
Assume:
- \( x = \) solution concentration 1
- \( y = \) solution concentration 2
- \( z = \) solution concentration 3

Mathematical models:
- \( 0.2x + 0.3y + 0.45z = 0.38 \) (equation 1)
- \( x + y + z = 10 \) (equation 2)

Determine the volume of the each solution?
Answer:
- \( 0.2x + 0.3y + 0.45z = 0.38 \times 10 = 3.8 \times 20 \)

Elimination:
- \( 4x + 6y + 9z = 76 \)
- \( x + y + z = 10 \)

Elimination:
- \( x + y + z = 10 \)
- \( y - 2x = 1 \)

Elimination:
- \( 2y + 5z = 36 \)
- \( 3y + 2z = 21 \)
Figure 3 shows the problem-solving process performed by S3. It appears that the subject did not continue the calculation process to obtain the result. After the interview section, the subject revealed that the time given to work on the problem was not enough, so he felt rushed to finish it. S3 expressed this situation in the following interview excerpt.

R: “Please re-read this question!”
S3: “A chemist mixes three glucose solutions with a concentration of 20%, 30%, and 45% to produce 10 L of a glucose solution at a concentration of 38%. If the 30% solution volume used is 1 L more than twice the 20% solution used, specify each solution’s volume”.
R: “Do you know the problem that must be solved?”
S3: “Yes, I know; determine the volume of solution with a concentration of 20%, 30%, and 45% to produce 10L concentration solution 38% with the provisions already known in the problem”.
R: “Could you explain your answer? (While showing the results of the S3’s work)”
S3: “I wrote an example to create a mathematical model. Then, I proceeded by converting one of the mathematical models into a more straightforward form. After that, I used a completion rule of linear equations system to get the result”.
R: “did you get the result?”
S3: “Not yet. I did not continue the calculations because I had not enough time. I was rushed because I needed more time to understand the problem at the beginning of the process”.

The interview excerpt shows that the problem-solving stages of S3 subjects have been excellent and structured. S3 could analyze the linear equation system's problems well and create mathematical models to find solutions. However, the S3 was unable to gain the final result due to limited working time. Furthermore, the analysis of student’s errors based on Newman Procedure is found in Table 3.

<table>
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<tr>
<td>Reading</td>
<td>It could not be investigated through the subject’s work results.</td>
<td>Read the problem thoroughly and smoothly.</td>
</tr>
<tr>
<td>Comprehension</td>
<td>The subject performed mischief, wrote what was known, and asked.</td>
<td>Explained the answer, what was known, and the problem, which is to determine the volume of each solution used.</td>
</tr>
<tr>
<td>Transformation</td>
<td>Subject wrote problem-solving strategy using a linear equations system, i.e., elimination and substitution.</td>
<td>The subject had difficulty in creating mathematical models</td>
</tr>
<tr>
<td>Process skills</td>
<td>• The subject did not complete mathematical operations.</td>
<td>The subject had a lack of understanding in the calculation process and a lack of time.</td>
</tr>
<tr>
<td>Process skills</td>
<td>• The subject did not gain the final result.</td>
<td>The result of the previous error so not asked.</td>
</tr>
<tr>
<td>Encoding</td>
<td>The subject did not write conclusions.</td>
<td></td>
</tr>
</tbody>
</table>

Table 3 describes in detail the work results and interview on S3. The participant’s main error was in the Process Skills section, i.e., the subject did not complete the mathematical
operation in the problem-solving steps. This error affects the following process: the subject did not conclude at the end of the answer (Saputri, 2019).

The error done by S3 was not very significant because the subject could understand the problem and determine the problem-solving strategy well. Nevertheless, the subject argued that the teacher should add the time allocation. The subject also explained that understanding the problem took a longer time than creating the mathematical model. Kamsurya (2019) revealed that students could not complete the overall problem-solving process due to longer time in the early stages of completion, resulting in students not getting the final answer. It is also supported by Mahdayani (2016) and Novferma (2016), who wrote that the difficulty factor outside of the cognitive factors experienced by students in mathematical problem-solving is that students feel anxious, rushed in work and claiming that they have insufficient time. Thus, an alternative solution that could be given to a high-ability subject is the teacher could provide problems as exercises that could improve the students' speed of counting and thoroughness in resolving a problem.

CONCLUSION

This study shows that low-level students' main errors took place at the comprehension stage. Students did not correctly implement the problem-solving procedure because they did not understand the problem given. In a medium-ability student, an error occurred at the transformation stage that students could understand the problem and compile what was required in the problem. However, they could not arrange a problem-solving strategy. The main error performed by high-ability students was less significant and happened in the process skills section. Students did not complete mathematical operations in the problem-solving steps because they had not enough time.

Recommendations from the results of this study to minimize students’ error in the problem-solving process are: (1) Low-ability students are required to read the question carefully, educators should emphasize the problem-solving procedure, and students should strengthen the prerequisite learning concept understanding in problem-solving; (2) Medium-ability students should focus on the emphasis and development of their skills in understanding the language of a problem and balanced with improving learning concept mastery and contextual training; and (3) High-ability students should conduct exercises that could improve their counting speed and thoroughness of the subject in resolving a problem.
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